6-2 Operations with Polynomials

Main Ideas

- Add and subtract polynomials.
- Multiply polynomials.

New Vocabulary

degree of a polynomial

GET READY for the Lesson

Shenequa has narrowed her choice for which college to attend. She is most interested in Coastal Carolina University, where the current year's tuition is $3430. Shenequa assumes that tuition will increase at a rate of 6% per year. You can use polynomials to represent the increasing tuition costs.

College Choices

College	Tuition
Allegheny College	$26,650
University of Maryland	$7821
Coastal Carolina University	$3430

Add and Subtract Polynomials If r represents the rate of increase of tuition, then the tuition for the second year will be $3430(1 + r)$. For the third year, it will be $3430(1 + r)^2$, or $3430r^2 + 6860r + 3430$ in expanded form. The **degree of a polynomial** is the degree of the monomial with the greatest degree. For example, the degree of this polynomial is 2.

Study Tip

Look Back

You can review **polynomials** in Lesson 1-1.

EXAMPLE Degree of a Polynomial

1 **Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial.**

a. $\frac{1}{6}x^3y^5 - 9x^4$

This expression is a polynomial because each term is a monomial. The degree of the first term is $3 + 5$ or 8, and the degree of the second term is 4. The degree of the polynomial is 8.

b. $x + \sqrt{x} + 5$

This expression is not a polynomial because $\sqrt{x}$ is not a monomial.

c. $x^{-2} + 3x^{-1} - 4$

This expression is not a polynomial because x^{-2} and x^{-1} are not monomials. $x^{-2} = \frac{1}{x^2}$ and $x^{-1} = \frac{1}{x}$. Monomials cannot contain variables in the denominator.

CHECK Your Progress

1A. $\frac{x}{y} + 3x^2$ **1B.** $x^5y + 9x^4y^3 - 2xy$

To *simplify* a polynomial means to perform the operations indicated and combine like terms.

EXAMPLE Simplify Polynomials

Study Tip

Alternate Methods

Notice that Example 2a uses a horizontal method and Example 2b uses a vertical method to simplify. Either method will yield a correct solution.

2 **Simplify each expression.**

a. $(3x^2 - 2x + 3) - (x^2 + 4x - 2)$

Remove parentheses and group like terms together.

$(3x^2 - 2x + 3) - (x^2 + 4x - 2)$

$= 3x^2 - 2x + 3 - x^2 - 4x + 2$ Distribute the -1.

$= (3x^2 - x^2) + (-2x - 4x) + (3 + 2)$ Group like terms.

$= 2x^2 - 6x + 5$ Combine like terms.

b. $(5x^2 - 4x + 1) + (-3x^2 + x - 3)$

Align like terms vertically and add.

$$\begin{array}{r} 5x^2 - 4x + 1 \\ (+)\ -3x^2 + x - 3 \\ \hline 2x^2 - 3x - 2 \end{array}$$

CHECK Your Progress

2A. $(-x^2 - 3x + 4) - (x^2 + 2x + 5)$ **2B.** $(3x^2 - 6) + (-x + 1)$

Multiply Polynomials You can use the Distributive Property to multiply polynomials.

EXAMPLE Simplify Using the Distributive Property

3 **Find $2x(7x^2 - 3x + 5)$.**

$2x(7x^2 - 3x + 5) = 2x(7x^2) + 2x(-3x) + 2x(5)$ Distributive Property

$= 14x^3 - 6x^2 + 10x$ Multiply the monomials.

CHECK Your Progress **Find each product.**

3A. $\frac{4}{3}x^2(6x^2 + 9x - 12)$ **3B.** $-2a(-3a^2 - 11a + 20)$

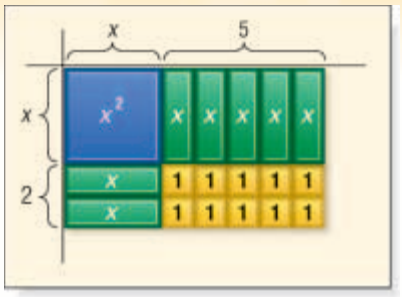
You can use algebra tiles to model the product of two binomials.

ALGEBRA LAB

Multiplying Binomials

Use algebra tiles to find the product of $x + 5$ and $x + 2$.

- Draw a 90° angle on your paper.
- Use an x tile and a 1 tile to mark off a length equal to $x + 5$ along the top.
- Use the tiles to mark off a length equal to $x + 2$ along the side.
- Draw lines to show the grid formed.
- Fill in the lines with the appropriate tiles to show the area product. The model shows the polynomial $x^2 + 7x + 10$.



The area of the rectangle is the product of its length and width.
So, $(x + 5)(x + 2) = x^2 + 7x + 10$.

EXAMPLE 4 Multiply Polynomials

Find $(n^2 + 6n - 2)(n + 4)$.

Method 1 Horizontally

$(n^2 + 6n - 2)(n + 4)$

$= n^2(n + 4) + 6n(n + 4) + (-2)(n + 4)$ Distributive Property

$= n^2 \cdot n + n^2 \cdot 4 + 6n \cdot n + 6n \cdot 4 + (-2) \cdot n + (-2) \cdot 4$ Distributive Property

$= n^3 + 4n^2 + 6n^2 + 24n - 2n - 8$ Multiply monomials.

$= n^3 + 10n^2 + 22n - 8$ Combine like terms.

Concepts in Motion
Animation
algebra2.com

Method 2 Vertically

$$\begin{array}{rr} & n^2 + 6n - 2 \\ (\times) & n + 4 \\ \hline & 4n^2 + 24n - 8 \\ n^3 + & 6n^2 - 2n \\ \hline n^3 + & 10n^2 + 22n - 8 \end{array}$$

CHECK Your Progress **Find each product.**

4A. $(x^2 + 4x + 16)(x - 4)$ **4B.** $(2x^2 - 4x + 5)(3x - 1)$

Online **Personal Tutor at algebra2.com**

CHECK Your Understanding

Example 1 (p. 320)

Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial.

1. $2a + 5b$ **2.** $\frac{1}{3}x^3 - 9y$ **3.** $\frac{mw^2 - 3}{nz^3 + 1}$

Simplify.

Examples 2–4 (pp. 321–322)

4. $(2a + 3b) + (8a - 5b)$ **5.** $(x^2 - 4x + 3) - (4x^2 + 3x - 5)$

6. $2x(3y + 9)$ **7.** $2p^2q(5pq - 3p^3q^2 + 4pq^4)$

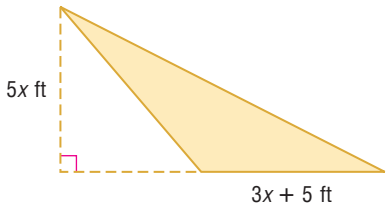
8. $(y - 10)(y + 7)$ **9.** $(x + 6)(x + 3)$

10. $(2z - 1)(2z + 1)$ **11.** $(2m - 3n)^2$

12. $(x + 1)(x^2 - 2x + 3)$ **13.** $(2x - 1)(x^2 - 4x + 4)$

Example 4 (p. 322)

14. GEOMETRY Find the area of the triangle.



Exercises

Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial.

15. $3z^2 - 5z + 11$ **16.** $x^3 - 9$ **17.** $\frac{6xy}{z} - \frac{3c}{d}$

18. $\sqrt{m - 5}$ **19.** $5x^2y^4 + x\sqrt{3}$ **20.** $\frac{4}{3}y^2 + \frac{5}{6}y^7$

HOMEWORK HELP

For Exercises	See Examples
15–20	1
21–24	2
25–28	3
29–36	4

Simplify.

21. $(3x^2 - x + 2) + (x^2 + 4x - 9)$

22. $(5y + 3y^2) + (-8y - 6y^2)$

23. $(9r^2 + 6r + 16) - (8r^2 + 7r + 10)$

24. $(7m^2 + 5m - 9) + (3m^2 - 6)$

25. $4b(cb - zd)$

26. $4a(3a^2 + b)$

27. $-5ab^2(-3a^2b + 6a^3b - 3a^4b^4)$

28. $2xy(3xy^3 - 4xy + 2y^4)$

29. $(p + 6)(p - 4)$

30. $(a + 6)(a + 3)$

31. $(b + 5)(b - 5)$

32. $(6 - z)(6 + z)$

33. $(3x + 8)(2x + 6)$

34. $(4y - 6)(2y + 7)$

35. $(3b - c)^3$

36. $(x^2 + xy + y^2)(x - y)$

37. PERSONAL FINANCE Toshiro has $850 to invest. He can invest in a savings account that has an annual interest rate of 1.7%, and he can invest in a money market account that pays about 3.5% per year. Write a polynomial to represent the amount of interest he will earn in 1 year if he invests x dollars in the savings account and the rest in the money market account.

E-SALES For Exercises 38 and 39, use the following information.
A small online retailer estimates that the cost, in dollars, associated with selling x units of a particular product is given by the expression $0.001x^2 + 5x + 500$. The revenue from selling x units is given by $10x$.

38. Write a polynomial to represent the profit generated by the product.

39. Find the profit from sales of 1850 units.

40. Simplify $(c^2 - 6cd - 2d^2) + (7c^2 - cd + 8d^2) - (-c^2 + 5cd - d^2)$.

41. Find the product of $x^2 + 6x - 5$ and $-3x + 2$.

Simplify.

42. $(4x^2 - 3y^2 + 5xy) - (8xy + 3y^2)$

43. $(10x^2 - 3xy + 4y^2) - (3x^2 + 5xy)$

44. $\frac{3}{4}x^2(8x + 12y - 16xy^2)$

45. $\frac{1}{2}a^3(4a - 6b + 8ab^4)$

46. $d^{-3}(d^5 - 2d^3 + d^{-1})$

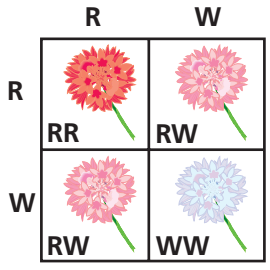
47. $x^{-3}y^2(yx^4 + y^{-1}x^3 + y^{-2}x^2)$

48. $(a^3 - b)(a^3 + b)$

49. $(m^2 - 5)(2m^2 + 3)$

50. $(x - 3y)^2$

51. $(1 + 4c)^2$



Real-World Link

Genetics

The possible genes of parents and offspring can be summarized in a *Punnett square*, such as the one above.

Source: *Biology: The Dynamics of Life*

52. GENETICS Suppose R and W represent two genes that a plant can inherit from its parents. The terms of the expansion of $(R + W)^2$ represent the possible pairings of the genes in the offspring. Write $(R + W)^2$ as a polynomial.

H.O.T. Problems

53. OPEN ENDED Write a polynomial of degree 5 that has three terms.

54. Which One Doesn't Belong? Identify the expression that does not belong with the other three. Explain your reasoning.

$3xy + 6x^2$ | $\frac{5}{x^2}$ | $x + 5$ | $5b + 11c - 9ad^2$

55. CHALLENGE What is the degree of the product of a polynomial of degree 8 and a polynomial of degree 6? Include an example to support your answer.

56. *Writing in Math* Use the information about tuition increases to explain how polynomials can be applied to financial situations. Include an explanation of how a polynomial can be applied to a situation with a fixed percent rate of increase and an explanation of how to use an expression and the 6% rate of increase to estimate Shenequa's tuition in the fourth year.

EXTRA PRACTICE
See pages 902, 931.

MathOnline
Self-Check Quiz at algebra2.com

STANDARDIZED TEST PRACTICE

57. ACT/SAT Which polynomial has degree 3?

A $x^3 + x^2 - 2x^4$

B $-2x^2 - 3x + 4$

C $x^2 + x + 12^3$

D $1 + x + x^3$

58. REVIEW $(-4x^2 + 2x + 3) - 3(2x^2 - 5x + 1) =$

F $2x^2$

G $-10x^2$

H $-10x^2 + 17x$

J $2x^2 + 17x$

Spiral Review

Simplify. Assume that no variable equals 0. (Lesson 6-1)

59. $(-4d^2)^3$ **60.** $5rt^2(2rt)^2$ **61.** $\frac{x^2yz^4}{xy^3z^2}$ **62.** $\left(\frac{3ab^2}{6a^2b}\right)^2$

Graph each inequality. (Lesson 5-8)

63. $y > x^2 - 4x + 6$ **64.** $y \leq -x^2 + 6x - 3$ **65.** $y < x^2 - 2x$

Determine whether each function has a maximum or a minimum value. Then find the maximum or minimum value of each function. (Lesson 5-1)

66. $f(x) = x^2 - 8x + 3$ **67.** $f(x) = -3x^2 - 18x + 5$ **68.** $f(x) = -7 + 4x^2$

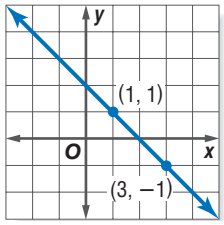
Use matrices A, B, C, and D to find the following. (Lesson 4-2)

$$A = \begin{bmatrix} -4 & 4 \\ 2 & -3 \\ 1 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 7 & 0 \\ 4 & 1 \\ 6 & -2 \end{bmatrix} \quad C = \begin{bmatrix} -4 & -5 \\ -3 & 1 \\ 2 & 3 \end{bmatrix} \quad D = \begin{bmatrix} 1 & -2 \\ 1 & -1 \\ -3 & 4 \end{bmatrix}$$

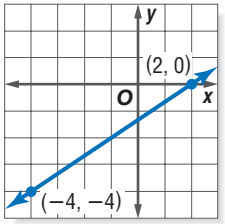
69. $A + D$ **70.** $B - C$ **71.** $3B - 2A$

Write an equation in slope-intercept form for each graph. (Lesson 2-4)

72.



73.



74. In 1990, 2,573,225 people attended St. Louis Cardinals home games. In 2004, the attendance was 3,048,427. What was the average annual rate of increase in attendance?

GET READY for the Next Lesson

PREREQUISITE SKILL Simplify. Assume that no variable equals 0. (Lesson 6-1)

75. $\frac{x^3}{x}$ **76.** $\frac{4y^5}{2y^2}$ **77.** $\frac{x^2y^3}{xy}$ **78.** $\frac{9a^3b}{3ab}$